Volume and Solid Modeling

CS418 Computer Graphics

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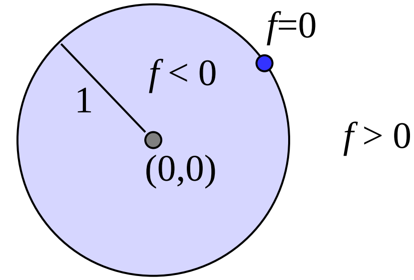
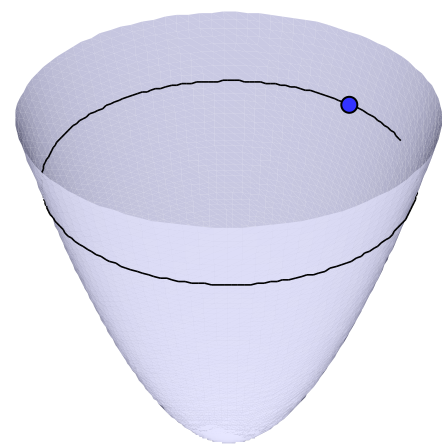
Implicit Surfaces

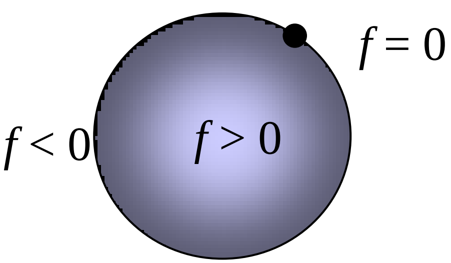


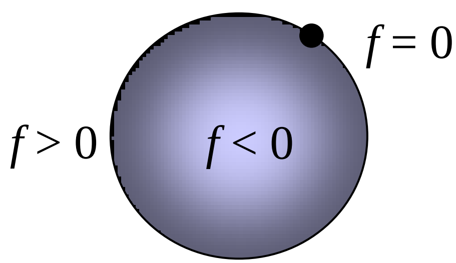
- Real function $f\,(x,y,z)$
- Classifies points in space
- Image synthesis (sometimes)
  - inside $f > 0$
  - outside $f < 0$
  - on the surface $f = 0$
- CAGD: inside $f < 0$, outside $f > 0$
- Surface $f^{-1}(0)$: Manifold if zero is a regular value of f

Circle example
$f(x,y) = x^2 + y^2 - 1$

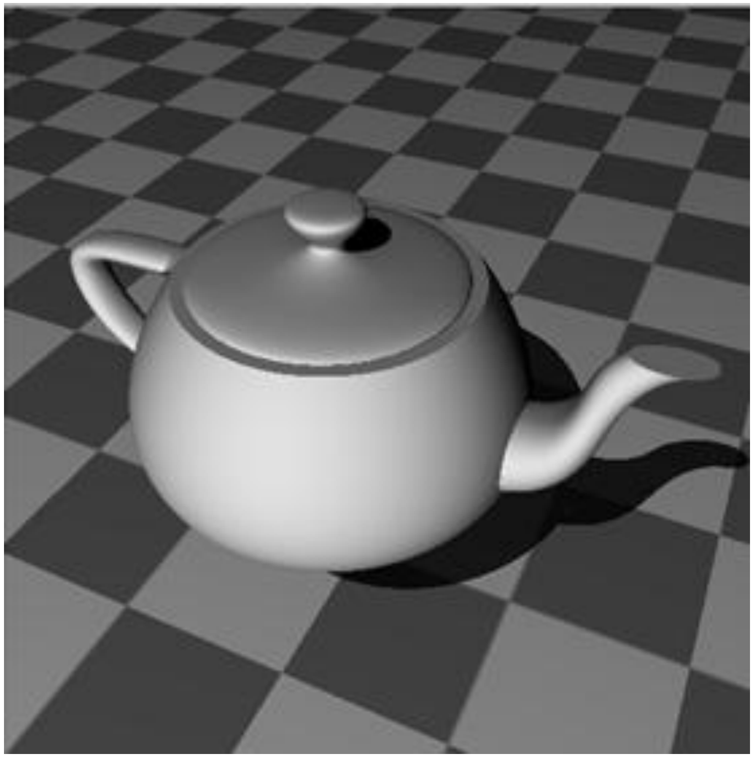


$z = f(x,y)$

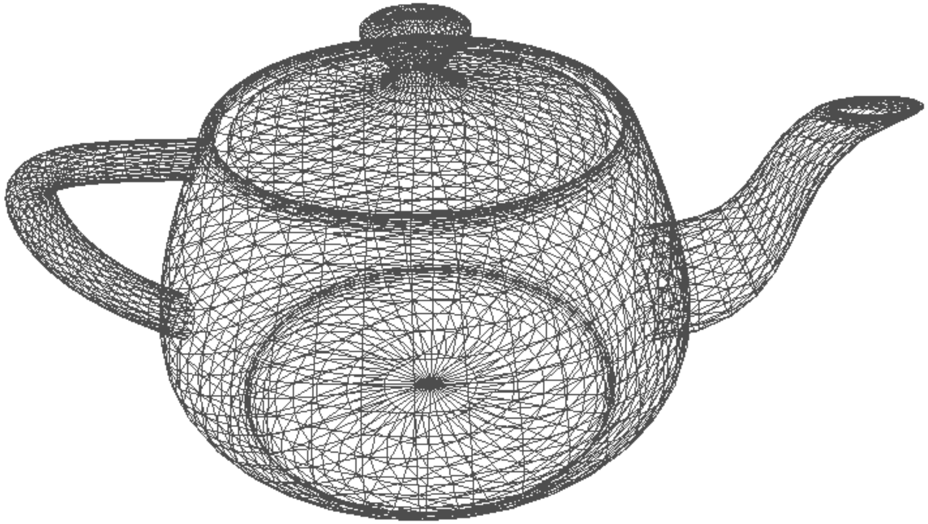




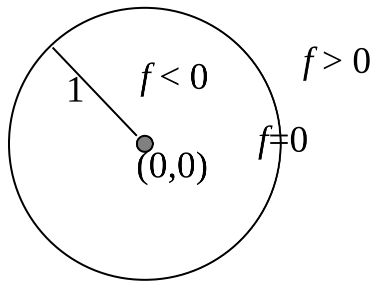
Why Use Implicits?



- v. polygons
  - smoother
  - compact, fewer higher-level primitives
  - harder to display in real time
- v. parametric patches
  - easier to blend
  - no topology problems
  - lower degree
  - harder to parameterize
  - easier to ray trace
  - well defined interior



Surface Normals

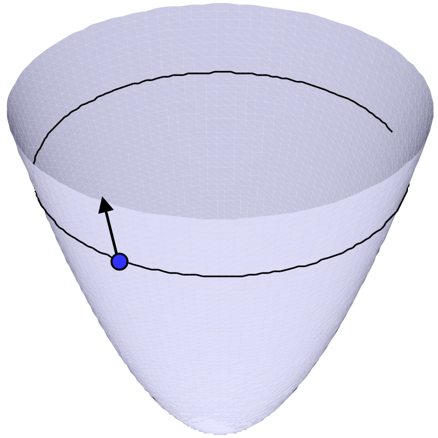


- Surface normal usually gradient of function

  $\nabla f(x,y,z) = (\delta f/\delta x, \delta f/\delta y, \delta f/\delta z)$

- Gradient not necessarily unit length
- Gradient points in direction of increasing f
  - Outward when $f < 0$ denotes interior
  - Inward when $f > 0$ denotes interior

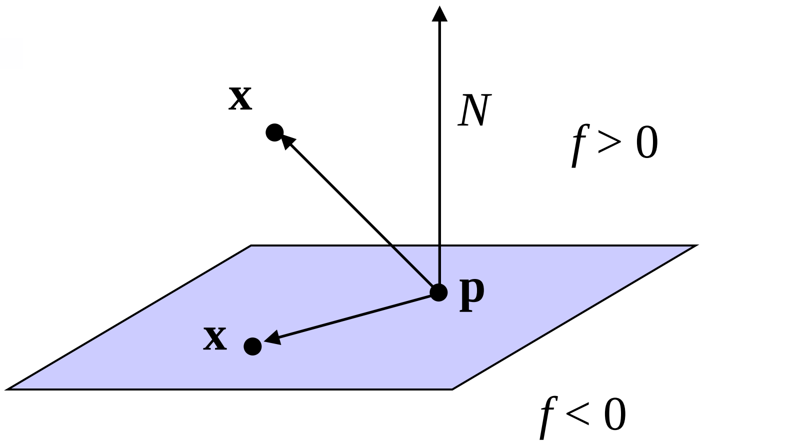
Circle example
$f(\mathbf{x}) = x^2 + y^2 - 1$
$\mathbf{x} = (x,y)$



$z = f(x,y)$

Plane

- Plane bounds half-space
- Specify plane with point **p** and normal N
- Points in plane **x** are perp. to normal N
- f is distance if $||N|| = 1$

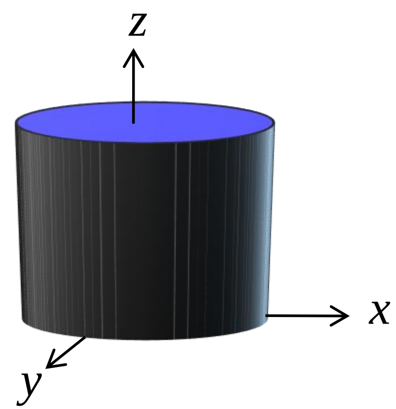


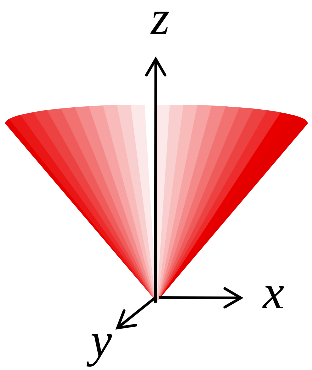
$$f(\mathbf{x}) = (\mathbf{x}-\mathbf{p})\cdot N$$

Quadrics

$$f(x,y,z) = Ax^2 + 2Bxy + 2Cxz + 2Dx + Ey^2 + 2Fyz + 2Gy + Hz^2 + 2Iz + J$$

- Sphere: f(x,y,z) = $x^2 + y^2 + z^2 - 1$
- Cylinder: f(x,y,z) = $x^2 + y^2 - 1$
- Cone: f(x,y,z) = $x^2 + y^2 - z^2$
- Paraboloid: $Ax^2 + Ey^2 - 2Iz = 0$





Homogeneous Quadrics

$f(\mathbf{x}) = \mathbf{x}^T Q\, \mathbf{x}$

$$f(x,y,z) = \begin{bmatrix} x & y & z & 1 \end{bmatrix} \begin{bmatrix} A & B & C & D \\ B & E & F & G \\ C & F & H & I \\ D & G & I & J \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Homogeneous coordinates

- $[x\ y\ z\ w]$
- Divide by w to find actual coords: $[x/w\ y/w\ z/w\ 1]$

Transforming quadrics

$\mathbf{x}\, Q\, \mathbf{x}^T = 0$, $\mathbf{x}' = \mathbf{x}T$

find Q' s.t. $\mathbf{x}' Q' \mathbf{x}'^T = 0$

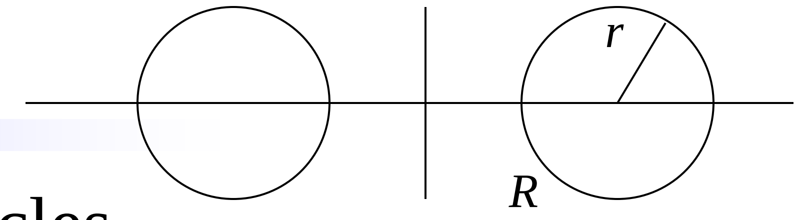
$\mathbf{x} = \mathbf{x}' T^*$ since x homo.

$\mathbf{x}' T^*\, Q\, (\mathbf{x}' T^*)^T = 0$

$\mathbf{x}' (T^*\, Q\, T^{*T})\, \mathbf{x}'^T = 0$

$Q' = T^*\, Q\, T^{*T}$

Torus



- Product of two implicit circles

  $(x - R)^2 + z^2 - r^2 = 0$

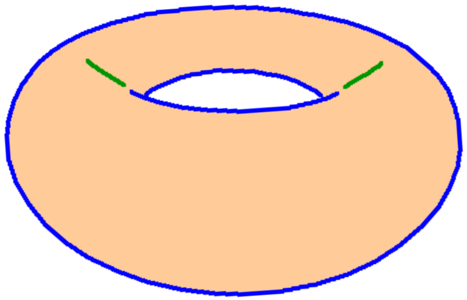
  $(x + R)^2 + z^2 - r^2 = 0$

  $((x - R)^2 + z^2 - r^2)((x + R)^2 + z^2 - r^2)$

  $(x^2 - Rx + R^2 + z^2 - r^2)\ (x^2 + Rx + R^2 + z^2 - r^2)$

  $x^4+2x^2z^2+z^4 - 2x^2r^2 - 2z^2r^2+r^4 + 2x^2R^2+2z^2R^2 - 2r^2R^2+R^4$

  $(x^2 + z^2 - r^2 - R^2)^2 + 4z^2R^2 - 4r^2R^2$



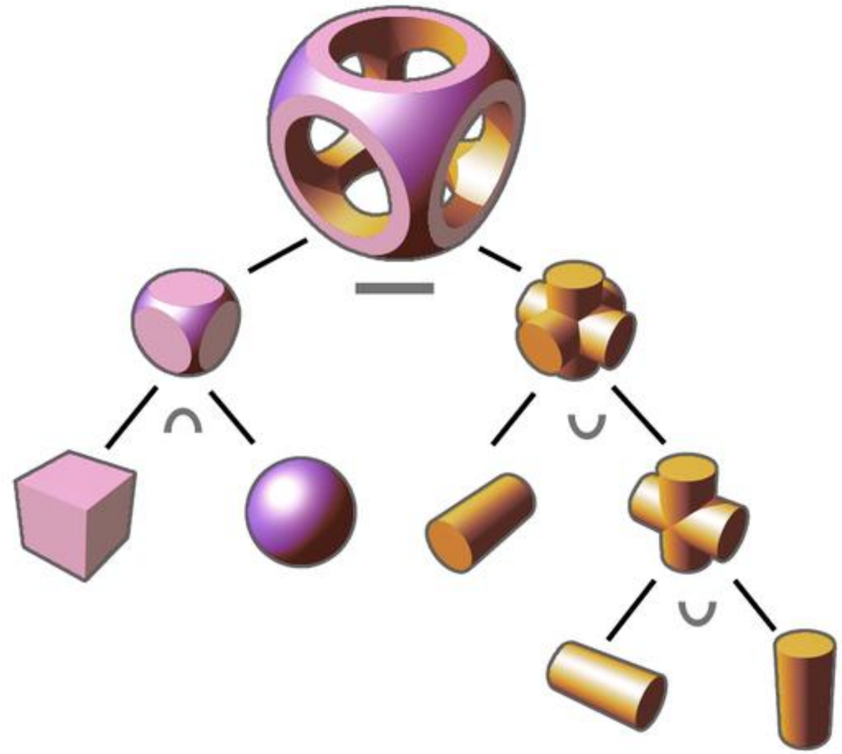
- Surface of rotation

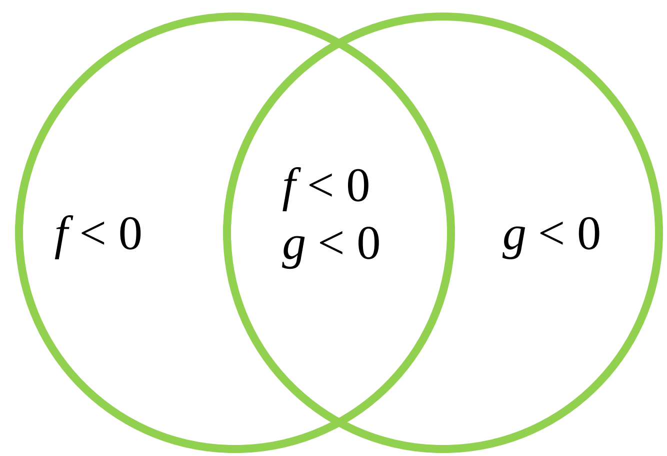
  replace x^2 with $x^2 + y^2$

  $f(x,y,z) = (x^2 + y^2 + z^2 - r^2 - R^2)^2 + 4R^2(z^2 - r^2)$

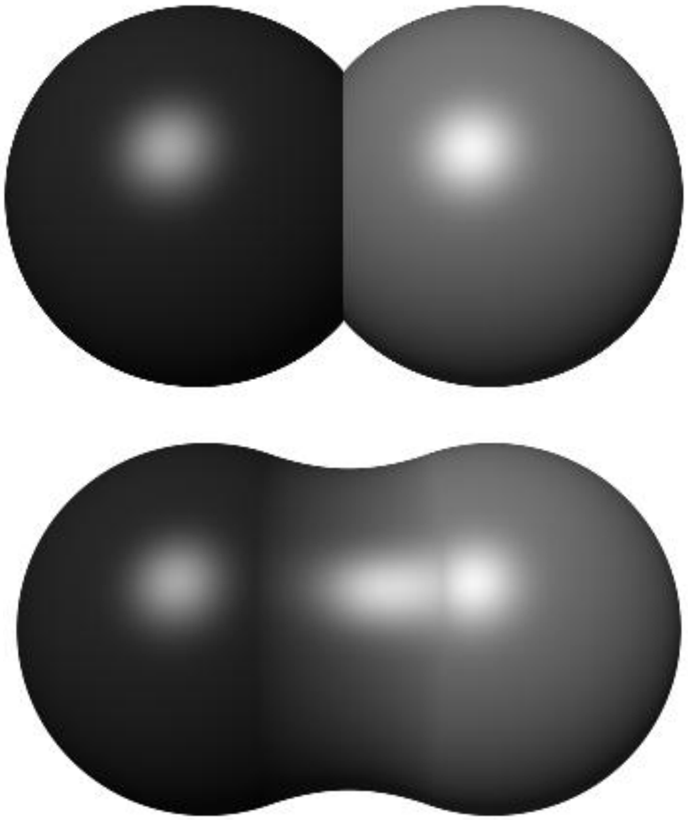
CSG

- Assume $f < 0$ inside
- CSG ops by min/max ops
  - Union: min f,g
  - Intersection: max f,g
  - Complement: $-f$
  - Subtraction: max $f,-g$
- Problem: C^1 discontinuity
- Can we smooth the blend crease?





Blobs



- Blinn TOG 1(3) 1982
- Sum of Gaussians

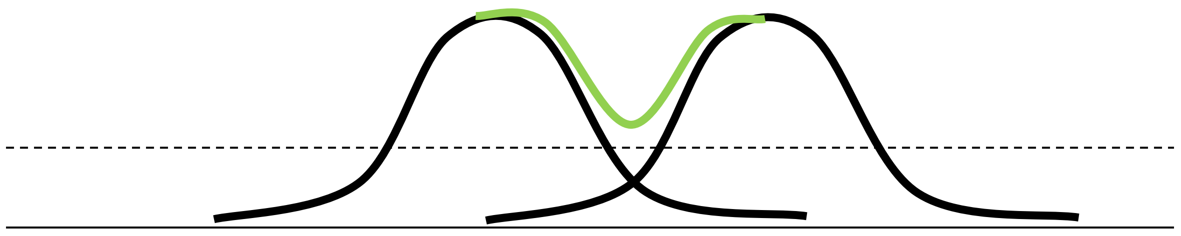
$$r_i^2(x,y,z) = x^2 + y^2 + z^2$$

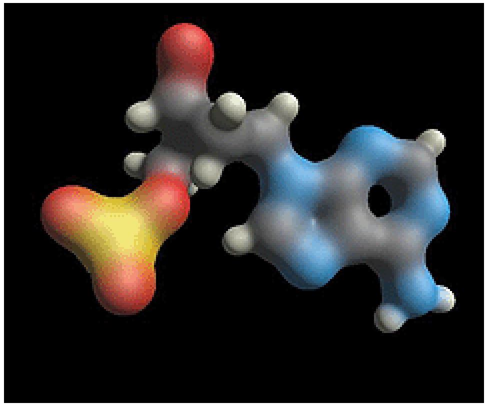
$$f(\mathbf{x}) = -1 + \sum \exp(-(B_i/R_i^2)r_i^2 + B_i)$$

B – blobbiness (positive)

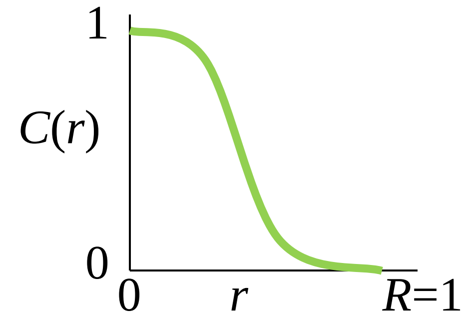
R – radius of blob at rest

r – radius function





Soft Objects



- Wyvill, McPheeters & Wyvill VC 86
- Exponential: too expensive, non-local
- Approximate $\exp(-r^2)$ with polynomial $C()$

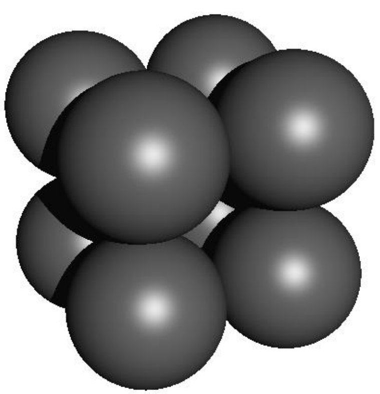
  $C(0) = 1$, $C(R) = 0$, $C'(0) = 0$, $C'(\mathrm{R}) = 0$

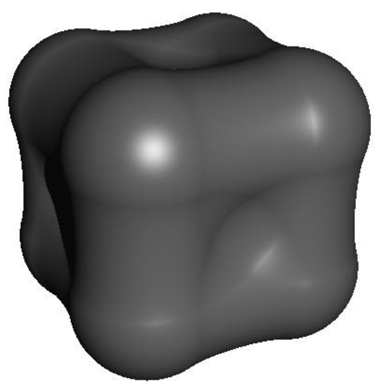
  $C(r^2) = -(4/9)r^6/R^6 + (17/9)r^4/R^4 - (22/9)r^2/R^2 + 1$

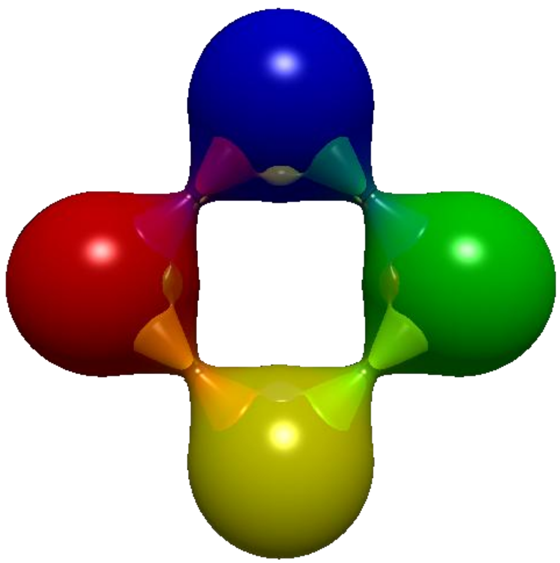
  $C(r) = 2r^3/R^3 - 3r^2/R^2 + 1$

  $C(r) = (1 - \mathrm{r}^2/R^2)^3$ (G^2 continuity)

  Pair of quadratics (metaballs)







Marching Cubes

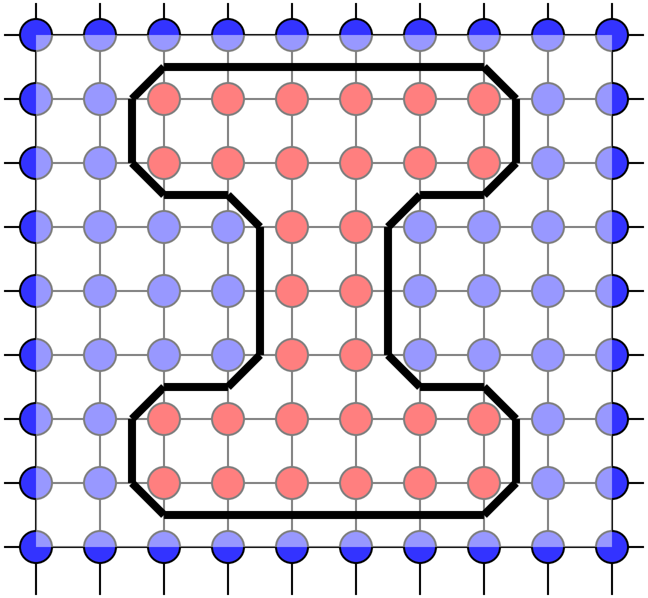
Read volume in two slices at a time

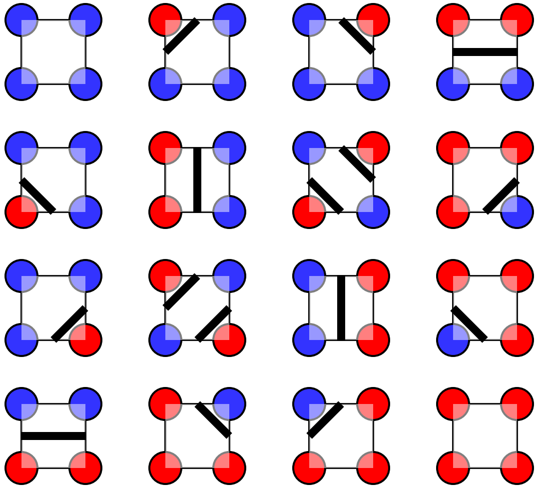
For each cubic cell

- Compute index using bitmask of vertices
- Output polygon(s) stored at index translated to cell position

End for

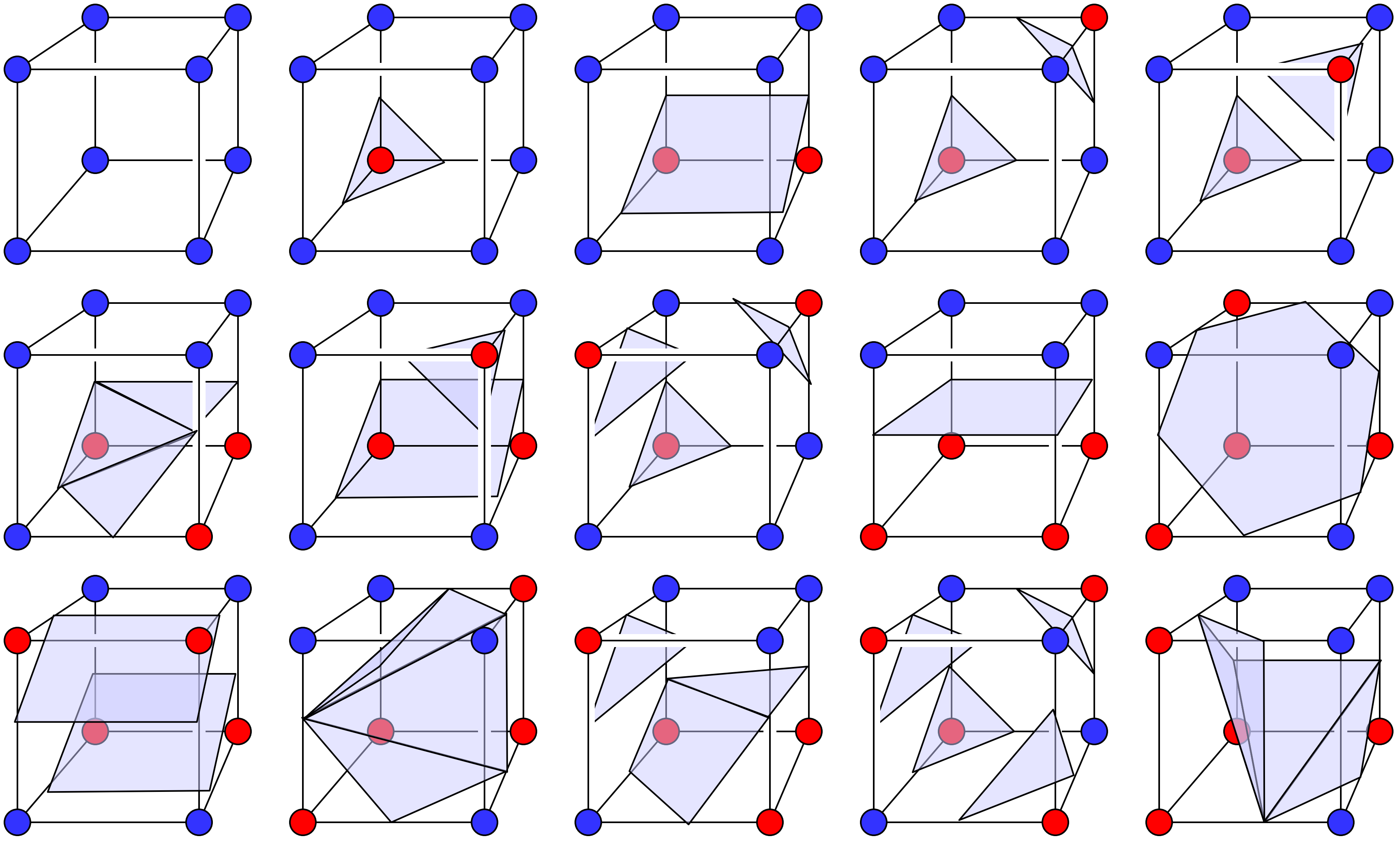
Remove last slice, add new slice, repeat





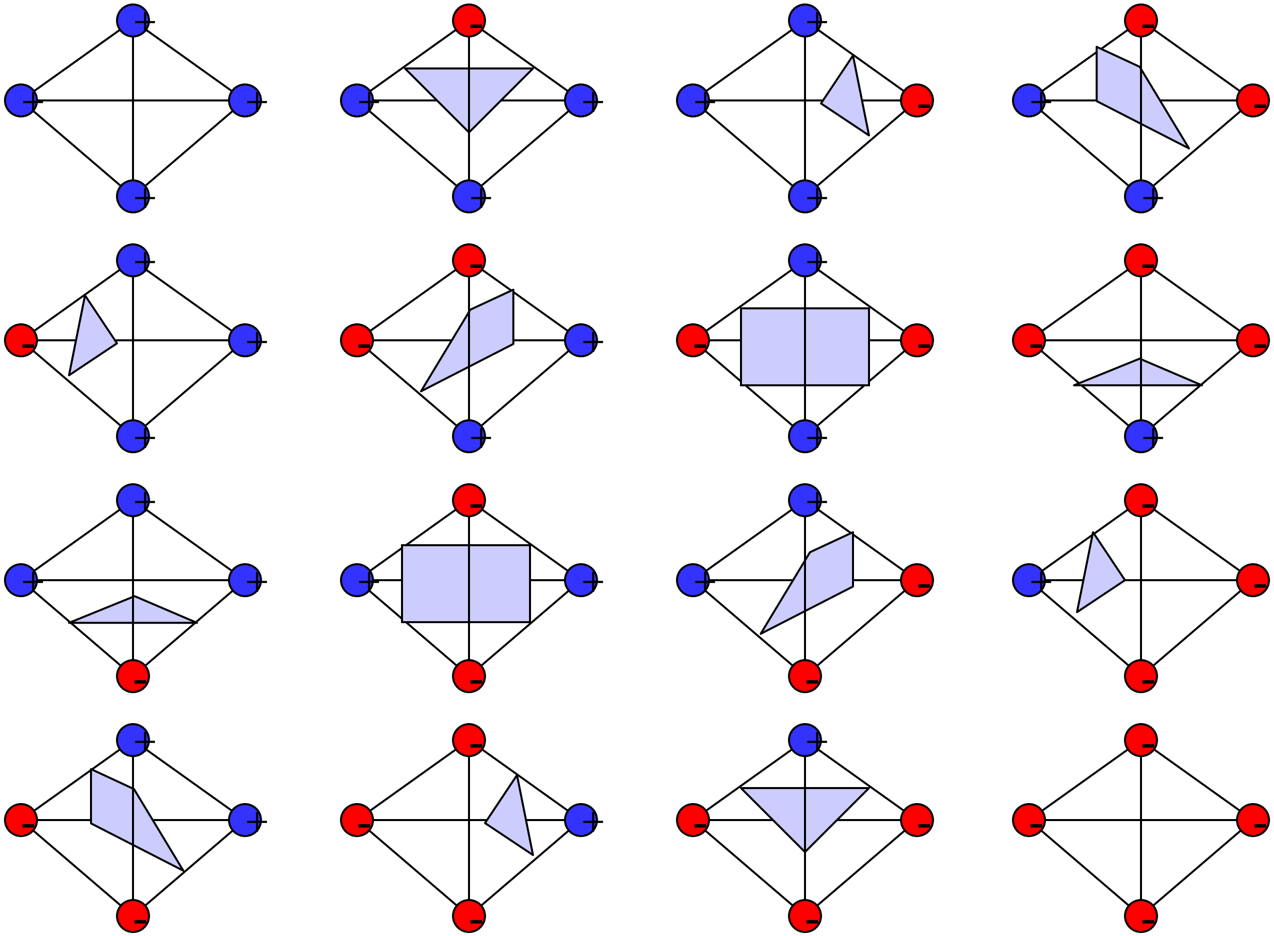
Marching Cube Cases

256 in all
15 modulo symmetry

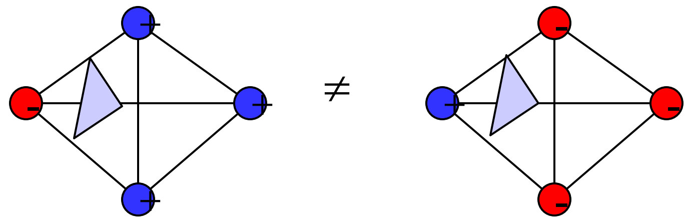


Marching Tet Cases

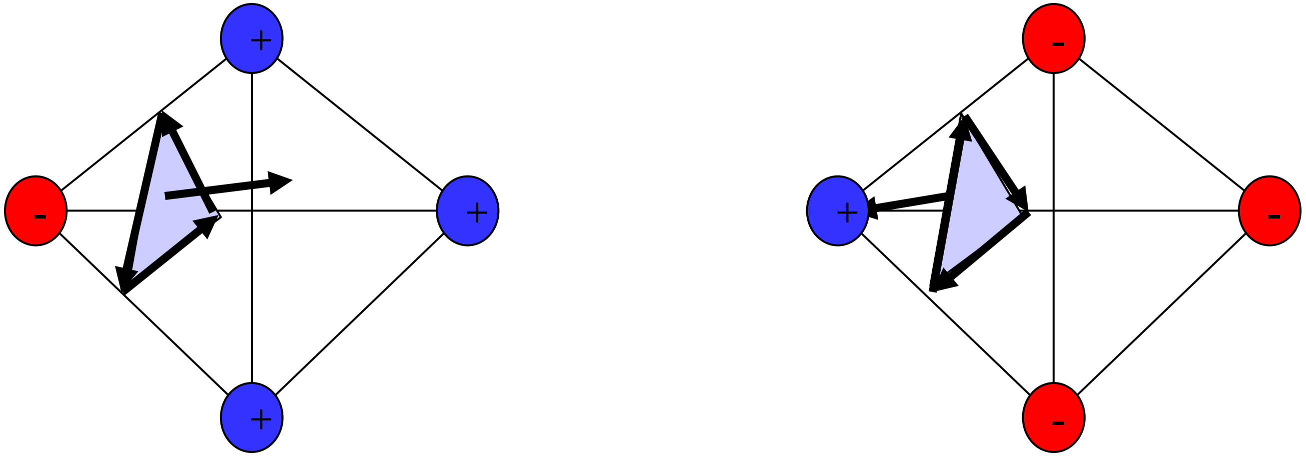
16 in all
3 modulo symmetry



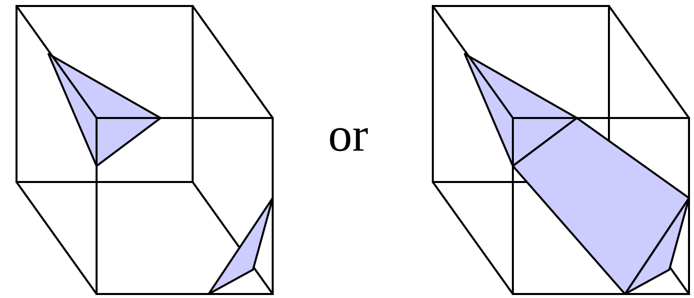
Orientation



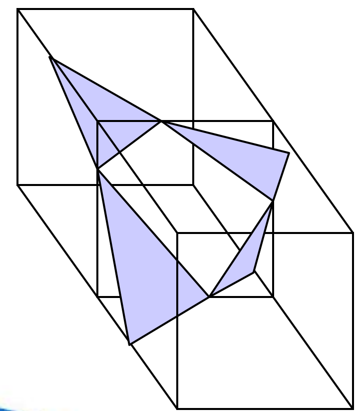
- Consistency allows polygons to be drawn with correct orientation
- Supports backface culling

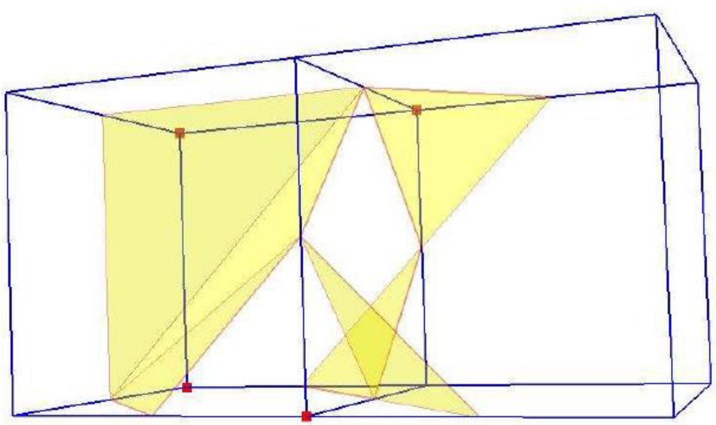


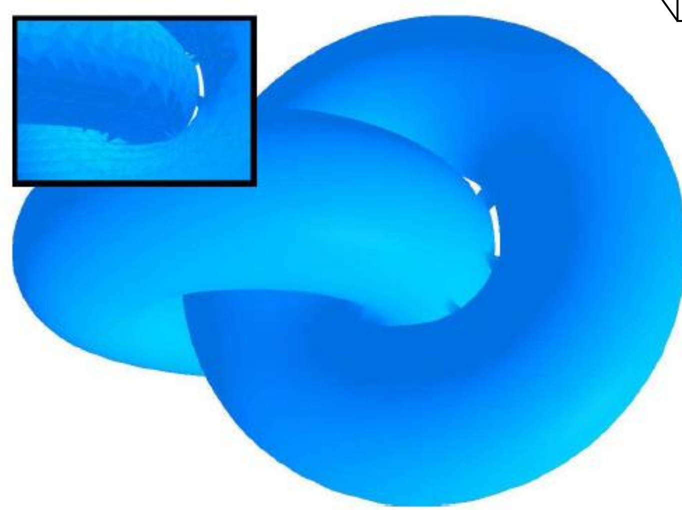
Problem: Ambiguity



- Some cell corner value configurations yield more than one consistent polygon
- Only for cubes, not tetrahedra (why?)
- In 3-D can yield holes in surface!
- How can we resolve these ambiguities?

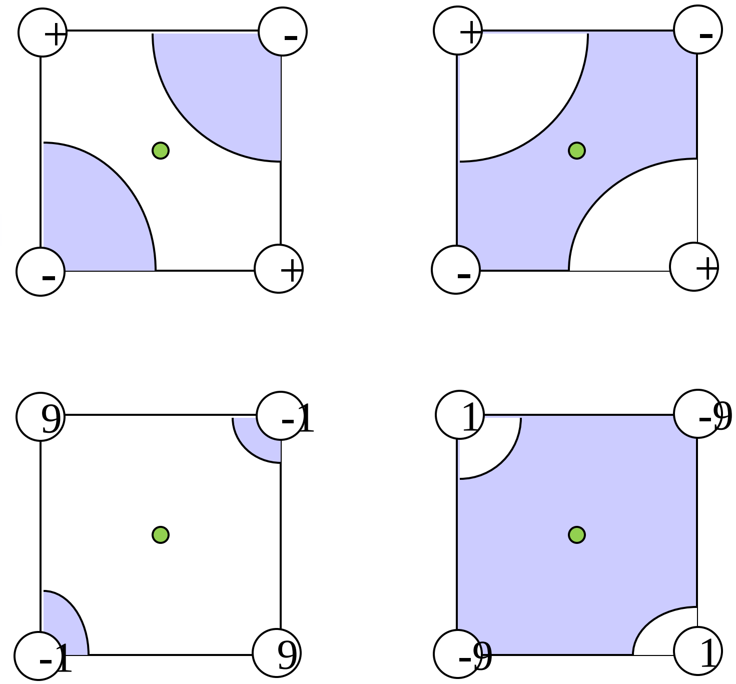






Examples from Lewiner et al., Efficient Implementation of Marching Cubes' Cases with Topological Guarantees. J. Graphics, GPU & Game Tools 8(2), 2003, pp. 1-15.

Problem: Ambiguity



- Some cell corner value configurations yield more than one consistent polygon
- Only for cubes, not tetrahedra (why?)
- In 3-D can yield holes in surface!
- How can we resolve these ambiguities?
- Topological Inference
  - Sample a point in the center of the ambiguous face
  - If data is discretely sampled, bilinearly interpolate samples

$$p(s,t) = (1-s)(1-t)\,a + s\,(1-t)\,b + (1-s)\,t\,c + s\,t\,d$$

Problem: Ambiguity

- Some cell corner value configurations yield more than one consistent polygon
- Only for cubes, not tetrahedra (why?)
- In 3-D can yield holes in surface!
- How can we resolve these ambiguities?
- Topological Inference
  - Sample a point in the center of the ambiguous face
  - If data is discretely sampled, bilinearly interpolate samples
- Preferred Polarity
  - Encode preference into table
  - ≡ cubes → tets
  - MC edges across neighboring faces must share direction to avoid cracks/holes